THE OPTIMAL BUDGET*

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Not for Quotation

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1. Introduction

What is the best way of allocating a given budget among the different functions of government? This paper is an attempt to answer that question in order to determine an "optimal budget". Such a budget is defined as the mix of public expenditures which maximizes the individual and social benefits that can be derived from a given total.

Three issues are investigated. The first is to find the optimal budget for an individual and to show the effect of income and education on the desired optimum. The second is to determine the budget mix which maximizes the collective benefits of all people in a jurisdiction. And the third is to compare this optimal social budget with the actual budget of a jurisdiction in order to observe whether the actual budget reflects the preferences of any particular income class.

The paper proceeds by first specifying the conditions for an optimal budget; then by finding the benefits that are associated with different expenditures. To do so, it deviates from traditional methodology. The analysis of public spending typically involves the attempt to find an "objective" value for the benefits of public programs; most cost-benefit studies follow this approach. This paper uses a different method: instead of trying to find objective measures of benefits it determines their subjective perception by different individuals. This is done through a theoretical model that can generate measures of benefit from referendum results, and by the empirical use of this model using data from Switzerland. The results then permit the determination of optimal budgets that correspond to different levels of income and education for individuals, and of the optimal budget of a jurisdiction.
II. Optimizing Criterion

An optimal budget is defined as the set of budget shares which maximizes the benefits that are derived from a given budget. If the budget size has a magnitude of $R$, and an expenditure of category $j$ is denoted by $V_j$, a budget share is equivalent to $V_j/R$. For an individual $i$, the satisfaction, or subjective benefit, that is generated by this budget allocation is $b_{ij}$, a function of the expenditure as well as of some other factors. Total satisfaction to the individual are the sum of these benefits, $B_i = \sum_j b_{ij}$. The question then is, what combination of budget shares will maximize the individual's total benefits. The answer is that total benefits are at a maximum where the marginal benefits due to each expenditure are equal. If marginal benefits were not equal, it would be possible to increase total benefits by shifting expenditures from budget items of lower to those of higher marginal benefits. This condition of equality of marginal benefits can be expressed, for a given budget $R$ in a time period, as

$$\frac{db_{i1}}{d(V_1/R)} = \frac{db_{i2}}{d(V_2/R)} = \ldots = \frac{db_{in}}{d(V_n/R)}$$

(1)

Another condition is that the budget shares must add up to one.

$$\sum_j \frac{V_j}{R} = 1$$

(2)

For society as a whole, satisfaction for each budget share is defined as the sum of individual benefits $S_j = \sum_i b_{ij}$

(3)

Again, total perceived benefits are highest where marginal societal benefits in each spending category $j$ are equalized, subject, as before, to the requirement that the sum of the shares is equal to one. This does not mean
that every individual's marginal benefits will be equal to everyone else's; only that the collective, or aggregate, marginal benefits are equal to each other for each spending category.

III. Benefit Model

Since the analysis is based on a concept of benefits that are perceived from a public expenditure, it is necessary to find a measure for this benefit. This presents a major problem since in the case of public goods, as distinguished from private goods, no market mechanism exists that would reveal the preferences of consumers by their willingness to pay. Therefore, an indirect method must be developed to find the perceived benefits for public expenditures. This paper uses as an alternative source of information for citizen preferences the voting behavior in public referenda. Referenda are useful because they reveal the desires of voters who are confronted with a real decision rather than with a hypothetical survey question, and where information about benefits, costs and disadvantages of the public good that is voted upon have been disseminated through public debate. On the other hand, there is a methodological problem with analyzing referenda; they normally reflect a single yes/no decision without an allowance for the intensity of the preference. This problem, which can make the preferences of a merely mildly interested group indistinguishable from those of a passionately concerned one, has hampered the previous use of referendum results. In this paper a method is used that can overcome this obstacle; it was developed by the author for the determination of demand functions for public goods.

The model is quite simple. One of its basic assumptions is that a voter tends to vote affirmatively for a proposal when he perceives its
benefits to him to be positive, and negatively if benefits are perceived to be negative.³ In Graph 1, points to the right of 0 on the horizontal axis represent positive benefits.

A second assumption is that within a narrowly defined population group, benefits will be normally distributed around some mean value Z.⁴ The distance Z is then a measure of mean benefits within the group. The larger Z, the larger is the positive (or negative) mean benefit. Thus, it is important to find the distance Z. How can this be done? The answer lies in the relationship between areas and distances under the normal distribution curve. Thus there will be an area Y that corresponds to 'yes' votes (those with high positive benefits), an area N of 'no' votes (those with high negative benefits), and an area A of 'abstaining' votes. The last group requires more explanation. In referenda, some people vote 'yes', some vote 'no', and still others do not vote at all. There is information in the actual voting, but also in the non-voting. While some people are habitual non-voters, others vote on issues that are important to them. In terms of Graph 1, there is a range to the left and right of 0, between S and -S, where positive or negative net benefits are small relative to the effort or the satisfaction of voting. This range can be termed the "indifference range", in which voters do not have enough stake in the outcome to go to the polls. It corresponds to related non-voting assumptions in the literature.⁵

The relative frequency of responses—yes, no, and abstain—can be represented by areas Y, N and A under the normal distribution curve. It is then possible to find the distance Z which represents the mean benefit to the group. Intuitively, Z is the relative distance that corresponds, for a normal distribution, to the shaded area in Graph 1. Given values for Y, N,
A, and S, one can determine $R$, and then find $Z$ by consulting the tables. 6

A more rigorous description of the method is given in the Appendix.

This information can be used to estimate the effect of certain factors on the perceived benefits. It stands to reason that the demographic characteristics of the voter will influence his interest and the benefits that he will associate with different types of public expenditure. Among
Figure 1
Distribution of Benefits

NUMBER OF PEOPLE

NET BENEFITS
Graph 1

Number of People

(Net Benefits)
table done elsewhere

Graph 1

number of people

(Net Benefits)
demographic characteristics, income and education are likely to have a strong impact since they tend to define the voter's socio-economic status. It is also likely that perceived benefits will be influenced by the absolute magnitude of the expenditure $V_j$. Where the total budget $R$ is given, this is equivalent to saying that benefits are also a function of the share $V_j/R$. These three factors, income, education, and size of expenditure, are then used to define a benefit function

$$b_{ij} = \alpha_j \frac{Y_i}{\bar{Y}} \rho_j \chi_j (V_j/R)^{\eta_j}$$

(4)

$Y_i/\bar{Y}$ is a measure of income relative to the median income in the jurisdiction (relative income is considered to be a more meaningful measure of socio-economic status than absolute income); $E_i$ is education; $V_j/R$ is the budget share; $\alpha, \rho, \chi$ and $\eta$ are parameters that must be estimated.

Into this equation we substitute for the budget share $V_j/R$ a measure of the individual's personal tax cost of that expenditure. The contribution $C_{ij}$ of the individual bears the same proportion to a budget expenditure $V_j$ as the individual's total tax payment $T_i$ bears to the total budget $R$, i.e.,

$$\frac{C_{ij}}{V_j} = \frac{T_i}{R}.$$  

The individual's total tax payment $T_i$ in turn is a function of income, $T_i = \varepsilon Y_i^\pi$. Therefore, the budget share can be expressed as

$$\frac{V_j}{R} = \frac{C_i}{\varepsilon Y_i^\pi}$$  

and this substituted into the benefit equation (3). After some algebraic rearrangement, that equation can then be expressed as (subscripts omitted)

$$b = a e^{-\eta Y (-\pi)} \left( \frac{Y}{\bar{Y}} \right)^{\rho \eta} \chi \varepsilon \eta$$

(4')
Given information about the independent variables, and about the corresponding \( b_{ij} \) -- which are generated by the referendum voting model -- the elasticities \( \rho, \chi, \) and \( \eta \) can be found by multiple regression analysis.

Marginal benefits are simply the derivative of total benefits. Differentiating equation (4)

\[
\frac{db}{d(V/R)} = \eta \alpha \left( \frac{Y}{V} \right)^\rho \cdot \frac{E^\chi(V/R)^{n-1}}{V}
\]  

(5)

with the parameters known, it is then possible to find the set of budget shares \( (V_j/R) \) for which marginal benefits are equal, and where the shares add up to one. Similarly, one can aggregate the marginal benefits for all individuals, \( S_j = \sum b_j \), for each share \( V_j/R \) and find the collective optimal budget at the mix where marginal collective benefits are equalized.

IV. Data

For empirical results, data for the Swiss Canton of Basel-City were used. There are a number of advantages in using that jurisdiction. Its voters frequently express, through referenda, their preferences for a large number of public expenditures. In its public goods provision, Basel comes close to a one-level government, since state and local functions are merged, and since national expenditures outside of defense are relatively small. Its compact area (14 square miles), which tends to reduce parochial influences of location, is also helpful. Basel is a highly developed yet middle-sized city with a long civic tradition and an international location. The preferences of its population are similar to those in many other western industrial societies.