Demand functions and the valuation of public goods

ELI M. NOAM*

1. Introduction

This paper describes a method to derive demand functions for public goods and to estimate their valuation by different groups of population. It uses the results of referenda and incorporates variations in non-voting into the model, obtaining the elasticities of preference intensities with respect to income, education and a variety of other factors. It then determines the value that high and low income groups attach to different public goods.

The difficulty of estimating the demand for public goods is a major problem for public resource allocation. Unlike private goods, public goods are not traded in a market, and no mechanism exists which relates quantities with their price. A number of attempts have been made to overcome this problem. One method is simply to ask people, by way of a survey or a controlled experiment, for their preferences (Bohm, 1972). The difficulty with this approach is that its application is time-consuming and expensive, and its reliability is inversely related to the subject's fondness for strategy. A related approach is therefore to refine the survey questions (Green and Laffont, 1976) or the cost shares associated with a public good (Tideman and Tullock, 1976) in a way that eliminates the incentive for such strategy. This approach impresses with its ingenuity, but its operational use does not seem easy.

A third method is to look at real life and to assume that reality is an expression of public desires (Borcharding and Deacon, 1972; Bergstrom and Goodman, 1973). Expenditures in different jurisdictions are then compared, and their relation to community characteristics such as income, education and tax rates is determined. The question remains whether the underlying

* This paper was supported by a research grant of the Center for Law and Economic Studies at Columbia Law School. Comments and suggestions on earlier drafts were made by Richard Musgrave, Nicolaus Tideman, Theodore Bergstrom and Melvin Hinich. Special thanks for their help go to Professor Peter Gilg of the Research Center for Swiss Politics at the University of Bern, to Oscar Wyss of the Statistical Bureau of the Canton of Basel-Stadt, and to Dr. Leopold Kohn. All responsibility rests with the author.

Graduate School of Business, Columbia University, Uris Hall, New York, NY 10027.
assumption that political decisions making accurately express public demand is reliable.

A fourth approach is to look at a mechanism available to register public preferences, namely voting results. In particular, the results of referenda are a valuable source of information because they reveal the preferences of voters who are confronted with real and publicly discussed expenditure questions rather than with hypothetical and only dimly understood issues. There is also normally no reason to believe that voters would vote contrary to their preferences for some strategic purposes. Recognizing these advantages, several studies of referenda on public finance issues were undertaken, but they involved a limited number of issues only and restricted themselves to statewide referenda (Deacon and Shapiro, 1975; Birdsall, 1965). And where more specific and local issues were analyzed, the studies are not concerned with demand functions but with the correlations of income with approval rates (Wilson and Banfield, 1965; Frey and Kohn, 1970).

In these studies an important problem remains unresolved: voting results indicate the direction of public preference, but they do not necessarily reflect its intensity.1 An issue may be mildly preferred or passionately desired by the same percentage of people; this will not be obvious from the voting results.

This paper suggests a method to overcome such problems by deriving a model by which measures of preference for public goods can be obtained. There preference measures can be analyzed for factors that influence them, and demand functions can then be found.

2. The model

Let us assume an identifiable and observable group of voters.2 Within the group, it is assumed that the net benefits $B$ that would accrue due to a referendum decision are distributed as a random variable with a normal distribution3 whose variance and mean are both unknown. $B$ is defined so that if benefits are positive about some indifference threshold $S$ a person votes ‘yes,’ and that if his perceived benefit is less than $-S$ he votes ‘no.’ Those whose benefits lie between $S$ and $-S$ are the abstainers, defined to exclude habitual non-voters. Formally,

\[
\text{Vote Yes} \quad \text{if} \quad B > S \\
\text{Vote No} \quad \text{if} \quad B < -S \\
\text{Abstain} \quad \text{if} \quad S < B \leq -S
\]

This distribution is represented in Figure 1, where abscissa points to the right of $O$ represent positive benefits, and beyond $S$, ‘yes’ voting. Points to the left of $-S$, ‘no’ voting. People in between are
abstainers. The percentage of 'yes' voters is $Y$, the percentage of 'no' voters is $N$, and the percentage of abstentions is $A$.

We want to find the mean benefit for the group. This amounts to determining the unknown mean $\mu$ of a normal distribution whose variance $\sigma$ is also not given, but where some areas $Y, N, A$ are known to exist, respectively, right of $S$, left of $-S$, and between $S$ and $-S$. Therefore

$$P\{B \leq S\} = N \quad P\{B \leq S\} = N + A$$

Standardizing these equations, one has

$$P\left\{Z \leq -\frac{S - \mu}{\sigma}\right\} = N \quad P\left\{Z \leq \frac{S - \mu}{\sigma}\right\} = N + A$$

In terms of $Z$, the cumulative distribution function of the standardized normal distribution

$$-\frac{S - \mu}{\sigma} = Z_N \quad \frac{S - \mu}{\sigma} = Z_{N+A}$$

so that the variance can be expressed as

$$\sigma = \frac{-2S}{Z_N - Z_{N+A}}$$

The mean of the distribution is therefore, after substitution,

$$\mu = -S - Z_N\sigma = S\left(\frac{Z_N + Z_{N+A}}{Z_N - Z_{N+A}}\right)$$
If the magnitudes of $N$ and $A$ are known, and with $Z_N$ and $Z_{N+A}$ that can be found from the tables, the value for the mean benefit $\mu$ can be readily calculated as a multiple of $S$.

$S$ need not be known if one assumes that it is the same for all groups; this assumption may also be relaxed or at least tested. To do so we use the reverse procedure on those issues for which the mean benefit $\mu$ of the normally distributed random variable $B$ is known, as well as the percentages $Y, N$, and $A$. From equations (3) we have

$$
\sigma = \frac{-2\mu}{Z_N + Z_{N+A}} \quad \text{and} \quad S = Z_N \frac{2\mu}{Z_N + Z_{N+A}} - \mu = \mu \left( \frac{Z_N - Z_{N+A}}{Z_N + Z_{N+A}} \right)
$$

When $N, A$, and $\mu$ are known, $S$ can be determined. It is now assumed that if $S$ is found to have relatively similar values for the several issues for which a mean benefit $\mu$ is known, then it is also similar for those issues for which $\mu$ is not known.

We have thus obtained a method for calculating the perceived benefits associated with different public goods. It is now possible to estimate factors that influence these perceived benefits, such as major demographic characteristics of the individual like income, age, number of children, and others. Let us assume that perceived benefits $B$ are functions of income $Y$, education $E$, age $A$, occupational status $O$, and family status $F$, such that

$$
B = f(Y, E, A, O, F) = \Omega Y^p E^q A^r O^s F^t
$$

(7)

equations of this form can be tested empirically for a variety of public goods. This is done in the following section.

3. Data

The empirical analysis is based on data and issues of referenda from Switzerland, since many public issues of even minor significance are decided in that country by the electorate directly. The canton of Baston-City is particularly instructive since its very small size reduces parochial influences of location. Basel, a highly developed middle-sized city with a long civic tradition and an international location should be illustrative for other western jurisdictions.

The main method of estimation involves a cross-section analysis across polling districts, where the demographic characteristics of the district are the independent variables, and the intensity of preference that is found through the model is the dependent variable.
Sources for the data are as follows: voting results for referenda by polling place are available in the official gazette. Demographic information on education, age, occupation and family structure is available from the Swiss national census and related publications. Income data is obtained from a market research survey. L, the active electorate, i.e., the electorate excluding habitual non-voters, is defined as the highest number of participants that voted at a polling place in a referendum concerning cantonal matters in each decade.

To estimate the threshold value, $S_i$, the procedure described earlier in equation (6) is used and issues are chosen for which the mean benefits $\mu_i$ can be estimated independently. The referenda issues that are used are votes on public utility charges and taxation. With the income distribution known, one can calculate the expected average financial consequences of an adoption of a tax, or of a charge vs. a tax proposal on a district, and use this value as $\mu_i$ in order to determine $S_i$ in (6). The resultant $S_i$ are all relatively similar to each other, with values in a band of 22-33 Sfr. in no discernable patterns. An average $S = 28$ Sfr. is therefore tentatively assumed to exist for other issues. A different $S$ would not change the elasticities.

An example will be given to illustrate the model, using the data for one subgroup of the population, the high-income neighborhood of Bruderholz. The first five issues in Table 1 are referenda for which the average monetaryeffect $\mu$ on different voters is either predicted in the official discussion of the issues (Ratschläge) which each voter receives, or, where appropriate, can be calculated from tax tables. Voting proportions are given in the first three columns; their cumulative $Z$ values are found from tables of the normal distribution, and $Z_N + Z_{N+2}/Z_N - Z_{N+A}$ are calculated in column four.

<table>
<thead>
<tr>
<th>Issue</th>
<th>$(Y/L)$</th>
<th>$(N/L)$</th>
<th>$(N + A)$</th>
<th>$Z_N + Z_{N+A}$</th>
<th>$\mu$</th>
<th>$S$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax reduction</td>
<td>52.3</td>
<td>35.0</td>
<td>47.7</td>
<td>1.369</td>
<td>40</td>
<td>29.2</td>
<td>183.38</td>
</tr>
<tr>
<td>Increased progressivity in tax rates</td>
<td>17.1</td>
<td>61.1</td>
<td>82.9</td>
<td>-1.835</td>
<td>-50</td>
<td>27.2</td>
<td>88.95</td>
</tr>
<tr>
<td>Gas utility rate change</td>
<td>32.2</td>
<td>11.6</td>
<td>67.8</td>
<td>.444</td>
<td>15</td>
<td>33.8</td>
<td>36.01</td>
</tr>
<tr>
<td>Water utility rate change</td>
<td>47.6</td>
<td>25.8</td>
<td>42.4</td>
<td>1.826</td>
<td>50</td>
<td>27.4</td>
<td>129.56</td>
</tr>
<tr>
<td>Water utility rate change II</td>
<td>44.0</td>
<td>10.9</td>
<td>56.0</td>
<td>.782</td>
<td>25</td>
<td>31.9</td>
<td>43.18</td>
</tr>
</tbody>
</table>
Multiplied with \( \mu \), according to equation (6), we obtain an estimate of \( S \). As can be seen, the values of \( S \) are quite similar to each other with an average of \( S_i = 29.8 \), suggesting that the general voting threshold is close to Sfr. 30 (about $17). Similar findings of stability at slightly different values are found for the other subgroups of the population.

If more referenda of this kind could be obtained, it would be possible to conclusively prove a stable threshold value, which would be an important finding in itself; from the limited observations for which \( \mu \) is available, we have an indication of support for the assumption of stability and for the use of the average \( S_i \) as the threshold for the particular subgroup. With \( S_i \) obtained, \( \mu \) can be calculated according to equation (5).

This procedure is repeated for all other subgroups, and regressions are run across the outcomes. Results for the different public goods follow in Table 2.

Table 2 lists elasticities for preferences in its left two columns. Almost all of these elasticities have a sign that is intuitively predictable. Even though positive income elasticities are more frequent, the several negative signs tend to disprove a notion of an 'income-trend' for a willingness for public expenditures, which Wilson and Banfield (1965) reported.

The results show that strong positive income elasticities are associated with law enforcement, foreign aid, airports, roads, educational expenditures and environmental controls. Negative income elasticities exist for all social services, particularly for unemployment compensation and for mass transit. As one would expect, theater and education have positive income elasticities, while those of sports are negative.

Elasticities with respect to education are mostly fairly small and have also the intuitively predictable signs, confirming voter rationality. These education elasticities are positive for expenditures on education itself, and for environmental protection, theater, mass transit, and most social programs. It is negative for law enforcement expenditures. The statistical significance of education elasticities is not overly high.

None of the other demographic factors shows consistent statistical importance; age is significant for road and mass transportation, but the coefficients are fairly small. Occupational status (here defined as self-employment) contributes to negative attitudes towards welfare and unemployment compensation. Family size affects the preference for mass transit, sports facilities, and education.

It is now interesting to estimate the absolute magnitude of these preferences. One way to interpret the meaning of such measures is to think of them as the maximum payment the individual would be willing to make to have the public good. Another interpretation is to think of the individual as being indifferent between having the public good, or having the stated sum of money. A larger sum would be preferable to the public good. Thus the value
Table 2. Elasticities of preference for public goods and services

<table>
<thead>
<tr>
<th></th>
<th>Income elasticity</th>
<th>Other significant elasticities</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment compensation</td>
<td>$-4.1283$ (1.9101)</td>
<td>Children $.0319$ (1.8268)</td>
<td>.9872</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Occupation $.0270$ (1.593)</td>
<td></td>
</tr>
<tr>
<td>Hospital</td>
<td>$-.0613$ (4.214)</td>
<td>Age $.0933$ (3.19211)</td>
<td>.6525</td>
</tr>
<tr>
<td>Welfare subsidies</td>
<td>$-.6529$ (.7165)</td>
<td>Occupation $-.2114$ (2.327)</td>
<td>.7922</td>
</tr>
<tr>
<td>Highways</td>
<td>$.6421$ (6.142)</td>
<td>Age $-.1231$ (2.3121)</td>
<td>.6129</td>
</tr>
<tr>
<td>Mass transit</td>
<td>$-.0742$ (2.2121)</td>
<td>Age $.2120$ (2.057)</td>
<td>.5703</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Children $.0278$ (1.9128)</td>
<td></td>
</tr>
<tr>
<td>Prosecutor</td>
<td>$2.9509$ (3.6136)</td>
<td></td>
<td>.8002</td>
</tr>
<tr>
<td>Prison</td>
<td>$2.0471$ (5.2218)</td>
<td>Occupation $.0124$ (1.8294)</td>
<td>.9162</td>
</tr>
<tr>
<td>Old-age home</td>
<td>$.0317$ (1.7212)</td>
<td>Age $.2915$ (2.5941)</td>
<td>.4672</td>
</tr>
<tr>
<td>Sports facilities</td>
<td>$-.1512$ (1.8632)</td>
<td>Children $.2033$ (2.191)</td>
<td>.5990</td>
</tr>
<tr>
<td>Museum</td>
<td>$1.5231$ (2.1327)</td>
<td>Education $.0829$ (4.1642)</td>
<td>.7219</td>
</tr>
<tr>
<td>Educational expenditures</td>
<td>$1.5826$ (1.7313)</td>
<td>Education $.2172$ (1.5272)</td>
<td>.7219</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Children $.6171$ (1.9597)</td>
<td></td>
</tr>
<tr>
<td>Foreign aid</td>
<td>$.7172$ (4.3191)</td>
<td></td>
<td>.8241</td>
</tr>
</tbody>
</table>

$t$-statistics in parentheses.

can also be seen as a schedule of 'bribes' to have a voter change his mind if he thinks that his vote makes the difference. These results depend by necessity on the magnitude of $S$. Because the calculated value of $S = 28$ is based on a few referenda only, the estimation should be considered a first approximation. As can be seen, high-income, high education individuals (upper 20%) have a
strong absolute valuation for education, parking spaces and law enforcement expenditures; their demand for roads, airport, and culture is positive but moderate in size. For social programs their demand is normally negative, but not strongly so. It does not equal the strong positive demand by the low income/low education group (lowest 20%) for these expenditure. For low income/low education people, not surprisingly, the most important issues involve services from which they benefit: public hospital, housing, and low income support. Nearly all other public goods are demanded positively – not surprising considering their low cost to poor people – but not very intensely.

4. Summary

This paper develops a method to derive measures of the demand for public goods. It takes the results of referenda and incorporates variation in non-voting into a model, which is used to calculate both the demand elasticities of the preference intensities with respect to a variety of demographic variables and the absolute magnitudes of such preference for a variety of public goods.

NOTES

1. Deacon and Shapiro (1975), use 'Utility' to overcome this problem, but only as an intermediate theoretical construct of their analysis.
2. Groups may be stratified by income, education, and other characteristics.
3. Normal distribution is a frequent assumption in much of the public choice literature on spatial electoral analysis. See e.g., Hinich and Ordeshook, (1970); Hinich, Ledyard and Ordeshook (1972). In the absence of information, the assumption of normality seems reasonable, particularly since we deal with the subjective perceptions of benefits.
4. For related discussion of abstentions, see Hinich and Ordeshook (1969), Hinich, Ledyard and Ordeshook (1972), Tolisson and Willett (1973).
5. The use of Basel was inspired by Frey and Kohn (1970).
6. Kantonsblatt, Basel-Stadt, on days following the referenda.
8. PROGNOS, Konsumpotential and Filialnetz, Untersuchung über die Investitionsplanung des ACV, for mean household income of 1965 in districts (Y) and for city (Y). Tables were made available by L. Kohn. The relative distribution is assumed to hold for other years as well.
9. A number of other definitions for the active electorate L were also investigated. For example, the highest participation in a federal referendum; the highest number of voters at a polling place at any election; the population within a district, minus the lowest abstention rate. The results of the analysis were fairly insensitive to the choice of those alternative definitions.
10. Referenda held 12/11/49; 1/19/50; 2/10/72; 7/12/75; 4/22/76.
11. PROGNOS, op. cit.
REFERENCES


