The Efficiency of Direct Democracy

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The paper investigates the Pareto efficiency of direct democracy and searches for the frequency with which the losing minority in referendum voting could compensate the majority and still be better off. A model is defined that permits the measuring of the intensity of preferences in a population, based on voting and abstention behavior. Using the model, an analysis of over 100 Swiss referenda reveals only a few instances in which the outcome of direct voting is inefficient. It seems that the political system evolves methods of trade-off that permit efficient outcomes in most cases. Criticism of direct democracy should therefore not rely on the primarily hypothetical objection of inefficiency.

A direct democracy is a political system in which policy decisions are made by the electorate itself and not by elected officials. Elements of such a system are found where referenda, plebiscites, propositions, or initiatives exist. Interest in direct democracy has grown considerably over the past years; in some states, such as California, the use of propositions has become an important form of policy determination, while in others—for example, New York—efforts are made to introduce more popular voting into state constitutions.

A main objection to referendum democracy is that it leads to outcomes that are not efficient: A majority will prevail even when its
potential gain is much smaller than the minority's loss. Both sides would be better off if the minority position were adopted, with the majority being compensated for its losses. In a parliamentary system, some form of trade-off or log-rolling could take place. But vote trading may not be possible in a direct democracy, because it is difficult to negotiate and impractical to enforce. Therefore, direct voting—in the absence of some system of intensity-weighted voting—is held to be subject to Pareto inefficiency (Tideman 1976).

While this view is justified on theoretical grounds, it is quite possible that in practice the political system has evolved ways to make such inefficiency unlikely. To judge direct democracy is then partly an empirical question. Can inefficient voting outcomes be observed in real politics? And if so, how frequent are they?

To answer these questions, this paper first defines an efficiency criterion to determine whether the majority has approved a position with negative net social benefits or rejected one with positive net social benefits. Next, in order to find these benefits, a method is described that permits their calculation from actual voting results. This calculation is possible, given knowledge of the number of people who voted yes and no and those who abstained, and given knowledge of the distribution of the relative benefits associated with certain referendum issues (public utility charges and taxes). After formulating the model, the paper analyzes empirically more than 100 Swiss referenda. The results show that inefficient outcomes occur but that they are quite infrequent. Evidently, the political system in the jurisdiction in question has evolved methods of trade-off that will normally permit the results of referenda to be Pareto efficient.

**Criterion**

Suppose that the passing of a given proposal will give a group $i$ with a membership $n_i$ a net benefit of $B_i$. Total benefits $T$ in the population will then be $T = \sum B_i n_i$. A positive $T$ means that the sum of positive benefits that are due to a referendum decision is larger than the sum of negative benefits. But it is possible that in the actual voting the proposal is defeated because the number of voters with negative benefits exceeds those with positive benefits. The excess of yes voters (those with positive benefits) over no voters can be expressed by $E = \sum [(B_i n_i) / |B_i|]$, where $B_i / |B_i|$ assigns a dimensionless sign to $n_i$.

The test for an inefficient vote is then to observe whether $T$ and $E$ have different signs; this means, for example, that total benefits are negative while the proposal is approved, that is, while there is an excess of voters with positive benefits. Thus the efficiency test becomes the sign of the ratio of $T$ and $E$, or
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\[ \rho = \frac{T}{E} = \frac{\sum B_i n_i}{\Sigma [(B_i n_i)/ |B_i|]} < 0. \]  

(1)

Where \( \rho \) is negative, the losing minority could have more than compensated the majority.\(^1\)

To test for efficiency according to this criterion, one must find the benefits that are associated with the referendum issue as they are perceived by different groups of voters.\(^2\) This presents a problem, because the approval percentages in an election are not necessarily an indicator of the magnitude of the benefits. Instead one must develop a different method. The next part of the paper describes a model that permits such measurement.

**Model**

We assume that the net perceived benefits \( B \) in a group \( i \) are distributed as a random variable with a normal distribution,\(^3\) with neither the variance nor the mean of this distribution known.

If the group has \( n_i \) people, we can approximate the benefits for that group as \( n_i \times \mu_i \), where \( \mu_i \) is the mean value of net benefits \( B \). The \( B \) is defined so that people vote yes if their benefits are positive above some indifference range \( S \) and vote no if their benefits are less than \( -S \). This is represented by figure 1, in which net benefits are shown on the abscissa and their frequency on the ordinate. Points to the right of \( O \) represent positive benefits, and those to the right of \( S \) represent yes voting. Points to the left of \( O \) represent negative benefits, and those to the left of \( -S \) represent no voting. People with benefits between \( S \) and \( -S \) are abstainers; they are defined to exclude habitual nonvoters and to consist of those voters who do not go to the polls for issues that are not significantly important to them. Formally, people vote when their benefit is \( |B| > |S| \) and do not vote when \( |B| < |S| \).\(^4\) Let \( Y \) be the percentage of yes voters, \( N \) be the percentage of no voters, and \( A \) be the percentage of abstentions; furthermore, let \( F \) be

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1. This efficiency test can also be interpreted as assigning weights to voters according to their stake in the outcome and comparing the result of such a weighted voting with that of the usual unweighted method.
2. Different weighting schemes are possible, for example, according to strictly monetary gains and losses; however, it seemed more reasonable to choose as weights the "perceived benefits" which include also the intangible benefits and costs that are associated with an issue.
3. Groups may be stratified by income, district of residence, and other characteristics. The empirical part of this study uses polling districts as groups. Districts correspond to fairly homogeneous neighborhoods.
4. This corresponds to similar nonvoting assumptions in the literature (Hinich and Ordeshook 1969; Hinich, Ledyard, and Ordeshook 1972; Tollison and Willett 1973).
the cumulative distribution function for $B$. Then $Y = 1 - F(S)$, $N = F(-S)$, and $A = 1 - N - A = 1 - F(S) - F(-S)$.

What we are interested in finding is the mean benefit for the group. This amounts to determining the unknown mean $\mu$ of a normal distribution whose variance $\sigma$ is also not given; however, the percentages (and hence the probability) of yes, no, and abstaining voters are known and are represented, respectively, by the areas to the right of $S$, left of $-S$, and between $S$ and $-S$. Therefore,

$$P(B \leq -S) = N \quad (2)$$

and

$$P(B \leq S) = N + A. \quad (3)$$

These equations, standardized, become

$$P\left(Z \leq \frac{-S - \mu}{\sigma}\right) = N \quad (2')$$

and

$$P\left(Z \leq \frac{S - \mu}{\sigma}\right) = N + A. \quad (3')$$

In terms of $Z$, the cumulative distribution function of the standardized normal distribution, $(-S - \mu)/\sigma = Z_N$ and $(S - \mu)/\sigma = Z_{N+A}$. Rewriting, we have the two equations $\mu = -S - Z_N \sigma$ and $\mu = S - Z_{N+A} \sigma$, so that $\sigma = -2S/(Z_N - Z_{N+A})$. The mean of the distribution can therefore be expressed, after substitution, as

$$\mu = -S - Z_N \sigma = S \left( \frac{Z_N + Z_{N+A}}{Z_N - Z_{N+A}} \right). \quad (4)$$
For group $i$, therefore, mean net benefits are

$$B_i = S_i \left( \frac{Z_{Ni} + Z_{Ni+Ai}}{Z_{Ni} - Z_{Ni+Ai}} \right).$$  \hspace{1cm} (4')$$

Equation (4') is used to calculate benefits in the text following.

If the magnitudes of $S$, $N$, and $A$ are known, and with $Z_N$ and $Z_{N+A}$ that can be found from the tables, the value for the mean benefit $\mu$ can be readily calculated. If the number of people in the group is $n_i$, the total net benefits for the group can thus be estimated as $n_i$ times the mean benefit for the group, $\mu_i$.

It is not necessary to know $S$ if one assumes that the voting threshold is the same for all groups, but this assumption may also be relaxed and different $S_i$ be estimated for different groups. For a group $i$, let there be an issue for which the mean benefit $\mu$ of the normally distributed random variable $B$ is known, as well as the percentages $Y$, $N$, and $A$. As before, $(-S - \mu)/\sigma = Z_N : S = -Z_N \sigma - \mu$ and $(S - \mu)/\sigma = Z_{N+A} : S = Z_{N+A} \sigma + \mu$. Therefore, $\sigma = -2\mu/(Z_N + Z_{N+A})$ and

$$S = Z_N \frac{2\mu}{Z_N + Z_{N+A}} - \mu = \mu \left( \frac{Z_N - Z_{N+A}}{Z_N + Z_{N+A}} \right).$$ \hspace{1cm} (5)

When $N$, $A$, and $\mu$ are known, $S$ can be determined. If $S$ is now found to be of a relatively similar value within group $i$ for the several issues for which a mean benefit $\mu$ is known, then it is assumed that such a threshold can also be used for those issues for which $\mu$ is not known. The efficiency test, which is the sign of the ratio $\rho$, does not depend on the magnitude of $S$, since with $B_i$ defined in (4'), and letting $\psi = (Z_{Ni} + Z_{Ni+Ai})/(Z_{Ni} - Z_{Ni+Ai})$, we have

$$\rho = \left| S \right| \frac{\sum |\psi_i| n_j}{\Sigma(|\psi_i| n_j)/(|\psi_i|)},$$ \hspace{1cm} (6)

and the sign of $\rho$ does not depend on $S$.

**Empirical Analysis**

The empirical analysis is based on data and referenda in the Swiss city-canton of Basel. In Switzerland more than in any other country, public questions of even minor significance are decided directly by the electorate, providing a wide range of issues. The analysis relied on 121 referenda from the canton Basel-Stadt during the years 1950–77.5 Voting results by polling places are available in the official

5 These include virtually all referenda for that time period. For seven referenda, data were unavailable or incomplete.
gazette; the corresponding districts are used as the subgroup $i$ for which mean perceived net benefits were calculated by equation (4') and over which total benefits $T$ were aggregated. Population figures $n_i$ are available for the districts from the Swiss national census (1976). The analytical use of the data requires a definition of an active electorate $L$ against which the percentage of active abstentions $A$ (as opposed to habitual nonvoting) can be measured. The definition that is chosen for the active electorate is the number of participants in the referendum with the highest voter turnout.

To estimate the threshold value, $S_i$, the procedure described earlier in the model is used and issues are chosen for which the mean benefits $\mu_i$ are known or can be calculated independently. The referenda issues that are used are votes on public utility charges and taxation. With the income distribution in each district known (PROGNOS 1972) one can calculate the expected average financial consequences of an adoption of a tax or of a charge versus a tax proposal on a district and use this value as $\mu_i$ in order to determine $S_i$. The resultant $S_i$'s are all relatively similar to each other, and we will assume therefore that $S_i$ is constant over $i$ and use its average value. As mentioned, its magnitude does not affect the sign of $p$. With the available data and parameters, it is now possible to calculate the mean net benefits $\mu_i$ and apply the efficiency test of equation (1) that was defined earlier.

Results are obtained for 121 cantonal referenda. Of these a total of six outcomes, or approximately 5 percent, were found to have the

6 See Kanton Basel-Stadt, Kantonsblatt on days following referenda.
7 The inner-city business district, an outlying small agricultural quarter (Bettingen), and a mixed outlying quarter (Rüehe) were omitted.
8 Referendum held October 20, 1974. A number of other definitions for the active electorate $L$ were also investigated, for example, the highest participation in a federal referendum, the highest number of voters at a polling place at any election, and the population within a district minus the lowest abstention rate. The results of the analysis were fairly insensitive to the choice of those alternative definitions.
10 The financial effects of these proposals are analyzed and made public in the official cantonal discussions and recommendations that are sent to each household prior to each vote (Kanton Basel-Stadt, Ratschläge, on dates preceding referenda). In the case of changes in the income-tax rates, the impact on different incomes is readily available; with the income distribution for each district known (PROGNOS 1972), the median impact can be determined. The cost of increased utility charges as an alternative to the use of tax revenue to cover utility deficits is given for the average-size household in the official recommendations listed above. (Adjustments according to average household size in the district alter these figures only insignificantly.) Substituting the parameters into eq. (5) for these issues, values for $S_i$ are found. They lie, for these issues and across all districts, in a fairly narrow band of SwFr 22–34, with no clear pattern such as an income trend. Hence a constant $S$ is assumed. Its actual magnitude, $S = SwFr 28$, is not in itself necessary for the calculation.
negative $\rho$ that indicates inefficiency." These issues involve public support for culture (theater) and for sports (two gymnasiums and a stadium), an airport extension, and facilities of the department of education; all of these projects should have been approved on the basis of efficiency, but they were narrowly defeated. In the remaining 95 percent of the referenda the majority decision had the same sign as total benefits; that is, the outcomes were efficient. Apparently, then, voting outcomes reflect the intensities of voters' preferences as well as the strict number of voters. The political system must have evolved mechanisms of trade-off that permit efficient outcomes in almost all cases.

The results of this study suggest that actual inefficiency in referendum voting exists at times but that its occurrence is fairly rare. Criticism of direct democracy should therefore not rely on an inefficiency argument, since it appears to be a primarily hypothetical objection.

References


-----. *Kantonsblatt*. Various numbers.


Additional Readings


" Theater subsidies (February 2, 1953; May 24, 1964); gymnasium (April 26, 1970); sports stadium (November 23, 1952); credit for airport extension (June 24, 1960); department of education, additional facilities (September 27, 1970).


